Edwards-Curve Digital Signature Algorithm (EdDSA)

Abstract

This document describes elliptic curve signature scheme Edwards-curve Digital Signature Algorithm (EdDSA). The algorithm is instantiated with recommended parameters for the edwards25519 and edwards448 curves. An example implementation and test vectors are provided.

Status of This Memo

This document is not an Internet Standards Track specification; it is published for informational purposes.

This document is a product of the Internet Research Task Force (IRTF). The IRTF publishes the results of Internet-related research and development activities. These results might not be suitable for deployment. This RFC represents the consensus of the Crypto Forum Research Group of the Internet Research Task Force (IRTF). Documents approved for publication by the IRSG are not a candidate for any level of Internet Standard; see Section 2 of RFC 7841.

Information about the current status of this document, any errata, and how to provide feedback on it may be obtained at http://www.rfc-editor.org/info/rfc8032.

Copyright Notice

Copyright (c) 2017 IETF Trust and the persons identified as the document authors. All rights reserved.

This document is subject to BCP 78 and the IETF Trust’s Legal Provisions Relating to IETF Documents (http://trustee.ietf.org/license-info) in effect on the date of publication of this document. Please review these documents carefully, as they describe your rights and restrictions with respect to this document.
Table of Contents

1.  Introduction ............................................... 3
2.  Notation and Conventions ................................. 4
3.  EdDSA Algorithm ........................................... 5
   3.1.  Encoding ............................................. 7
   3.2.  Keys ............................................... 7
   3.3.  Sign ................................................. 8
   3.4.  Verify .............................................. 8
4.  PureEdDSA, HashEdDSA, and Naming ....................... 8
5.  EdDSA Instances ........................................... 9
   5.1.  Ed25519ph, Ed25519ctx, and Ed25519 ............... 9
      5.1.1.  Modular Arithmetic ........................... 10
      5.1.2.  Encoding .................................... 10
      5.1.3.  Decoding .................................... 11
      5.1.4.  Point Addition ............................... 11
      5.1.5.  Key Generation ............................... 13
      5.1.6.  Sign ......................................... 13
      5.1.7.  Verify ...................................... 14
   5.2.  Ed448ph and Ed448 .................................. 15
      5.2.1.  Modular Arithmetic ........................... 16
      5.2.2.  Encoding .................................... 16
      5.2.3.  Decoding .................................... 16
      5.2.4.  Point Addition ............................... 17
      5.2.5.  Key Generation ............................... 18
      5.2.6.  Sign ......................................... 19
      5.2.7.  Verify ...................................... 19
6.  Ed25519 Python Illustration ............................ 20
7.  Test Vectors ............................................. 23
   7.1.  Test Vectors for Ed25519 ......................... 24
   7.2.  Test Vectors for Ed25519ctx ....................... 27
   7.3.  Test Vectors for Ed25519ph ....................... 30
   7.4.  Test Vectors for Ed448 ............................. 30
   7.5.  Test Vectors for Ed448ph ......................... 38
8.  Security Considerations .................................. 40
   8.1.  Side-Channel Leaks ................................ 40
   8.2.  Randomness Considerations ......................... 40
   8.3.  Use of Contexts ................................... 41
   8.4.  Signature Malleability ........................... 41
   8.5.  Choice of Signature Primitive .................... 41
   8.6.  Mixing Different Prehashes ......................... 42
   8.7.  Signing Large Amounts of Data at Once ............ 42
   8.8.  Multiplication by Cofactor in Verification ....... 43
   8.9.  Use of SHAKE256 as a Hash Function ............... 43
9.  References ............................................... 43
   9.1.  Normative References ............................. 43
   9.2.  Informative References ............................ 44
The Edwards-curve Digital Signature Algorithm (EdDSA) is a variant of Schnorr’s signature system with (possibly twisted) Edwards curves. EdDSA needs to be instantiated with certain parameters, and this document describes some recommended variants.

To facilitate adoption of EdDSA in the Internet community, this document describes the signature scheme in an implementation-oriented way and provides sample code and test vectors.

The advantages with EdDSA are as follows:

1. EdDSA provides high performance on a variety of platforms;
2. The use of a unique random number for each signature is not required;
3. It is more resilient to side-channel attacks;
4. EdDSA uses small public keys (32 or 57 bytes) and signatures (64 or 114 bytes) for Ed25519 and Ed448, respectively;
5. The formulas are "complete", i.e., they are valid for all points on the curve, with no exceptions. This obviates the need for EdDSA to perform expensive point validation on untrusted public values; and
6. EdDSA provides collision resilience, meaning that hash-function collisions do not break this system (only holds for PureEdDSA).

The original EdDSA paper [EDDSA] and the generalized version described in "EdDSA for more curves" [EDDSA2] provide further background. RFC 7748 [RFC7748] discusses specific curves, including Curve25519 [CURVE25519] and Ed448-Goldilocks [ED448].

Ed25519 is intended to operate at around the 128-bit security level and Ed448 at around the 224-bit security level. A sufficiently large quantum computer would be able to break both. Reasonable projections of the abilities of classical computers conclude that Ed25519 is perfectly safe. Ed448 is provided for those applications with relaxed performance requirements and where there is a desire to hedge against analytical attacks on elliptic curves.
2. Notation and Conventions

The following notation is used throughout the document:

- \( p \) denotes the prime number defining the underlying field
- \( \text{GF}(p) \) finite field with \( p \) elements
- \( x^y \) \( x \) multiplied by itself \( y \) times
- \( B \) generator of the group or subgroup of interest
- \( [n]X \) \( X \) added to itself \( n \) times
- \( h[i] \) \( i \)'th octet of octet string
- \( h_i \) \( i \)'th bit of \( h \)
- \( a || b \) (bit-)string \( a \) concatenated with (bit-)string \( b \)
- \( a \leq b \) \( a \) is less than or equal to \( b \)
- \( a \geq b \) \( a \) is greater than or equal to \( b \)
- \( i+j \) sum of \( i \) and \( j \)
- \( i\cdot j \) multiplication of \( i \) and \( j \)
- \( i-j \) subtraction of \( j \) from \( i \)
- \( i/j \) division of \( i \) by \( j \)
- \( i \times j \) Cartesian product of \( i \) and \( j \)
- \( (u,v) \) elliptic curve point with x-coordinate \( u \) and y-coordinate \( v \)
- \( \text{SHAKE256}(x, y) \) the \( y \) first octets of SHAKE256 [FIPS202] output for input \( x \)
- \( \text{OCTET}(x) \) the octet with value \( x \)
- \( \text{OLEN}(x) \) the number of octets in string \( x \)
RFC 8032  EdDSa: Ed25519 and Ed448  January 2017

dom2(x, y) The blank octet string when signing or verifying Ed25519. Otherwise, the octet string: "SigEd25519 no Ed25519 collisions" || octet(x) || octet(OLEN(y)) || y, where x is in range 0-255 and y is an octet string of at most 255 octets. "SigEd25519 no Ed25519 collisions" is in ASCII (32 octets).

dom4(x, y) The octet string "SigEd448" || octet(x) || octet(OLEN(y)) || y, where x is in range 0-255 and y is an octet string of at most 255 octets. "SigEd448" is in ASCII (8 octets).

Parentheses (i.e., '(' and ')') are used to group expressions, in order to avoid having the description depend on a binding order between operators.

Bit strings are converted to octet strings by taking bits from left to right, packing those from the least significant bit of each octet to the most significant bit, and moving to the next octet when each octet fills up. The conversion from octet string to bit string is the reverse of this process; for example, the 16-bit bit string

\[ b_0 \ b_1 \ b_2 \ b_3 \ b_4 \ b_5 \ b_6 \ b_7 \ b_8 \ b_9 \ b_{10} \ b_{11} \ b_{12} \ b_{13} \ b_{14} \ b_{15} \]

is converted into two octets x0 and x1 (in this order) as

\[ x_0 = b_7*128+b_6*64+b_5*32+b_4*16+b_3*8+b_2*4+b_1*2+b_0 \]
\[ x_1 = b_{15}*128+b_{14}*64+b_{13}*32+b_{12}*16+b_{11}*8+b_{10}*4+b_{9}*2+b_8 \]

Little-endian encoding into bits places bits from left to right and from least significant to most significant. If combined with bit-string-to-octet-string conversion defined above, this results in little-endian encoding into octets (if length is not a multiple of 8, the most significant bits of the last octet remain unused).

The key words "MUST", "MUST NOT", "REQUIRED", "SHALL", "SHALL NOT", "SHOULD", "SHOULD NOT", "RECOMMENDED", "MAY", and "OPTIONAL" in this document are to be interpreted as described in [RFC2119].

3. EdDSa Algorithm

EdDSa is a digital signature system with 11 parameters.

The generic EdDSa digital signature system with its 11 input parameters is not intended to be implemented directly. Choosing parameters is critical for secure and efficient operation. Instead, you would implement a particular parameter choice for EdDSa (such as
Ed25519 or Ed448), sometimes slightly generalized to achieve code reuse to cover Ed25519 and Ed448.

Therefore, a precise explanation of the generic EdDSA is thus not particularly useful for implementers. For background and completeness, a succinct description of the generic EdDSA algorithm is given here.

The definition of some parameters, such as n and c, may help to explain some steps of the algorithm that are not intuitive.

This description closely follows [EDDSA2].

EdDSA has 11 parameters:

1. An odd prime power p. EdDSA uses an elliptic curve over the finite field GF(p).

2. An integer b with $2^{(b-1)} > p$. EdDSA public keys have exactly b bits, and EdDSA signatures have exactly $2*b$ bits. b is recommended to be a multiple of 8, so public key and signature lengths are an integral number of octets.

3. A (b-1)-bit encoding of elements of the finite field GF(p).

4. A cryptographic hash function H producing $2*b$-bit output. Conservative hash functions (i.e., hash functions where it is infeasible to create collisions) are recommended and do not have much impact on the total cost of EdDSA.

5. An integer c that is 2 or 3. Secret EdDSA scalars are multiples of $2^c$. The integer c is the base-2 logarithm of the so-called cofactor.

6. An integer n with $c <= n < b$. Secret EdDSA scalars have exactly $n + 1$ bits, with the top bit (the $2^n$ position) always set and the bottom c bits always cleared.

7. A non-square element d of GF(p). The usual recommendation is to take it as the value nearest to zero that gives an acceptable curve.

8. A non-zero square element a of GF(p). The usual recommendation for best performance is $a = -1$ if $p \mod 4 = 1$, and $a = 1$ if $p \mod 4 = 3$.

9. An element $B \neq (0,1)$ of the set $E = \{ (x, y) \text{ is a member of} \ GF(p) \times GF(p) \text{ such that} \ a \cdot x^2 + y^2 = 1 + d \cdot x^2 * y^2 \}$. 
10. An odd prime L such that \([L]B = 0\) and \(2^c \cdot L = \#E\). The number \(\#E\) (the number of points on the curve) is part of the standard data provided for an elliptic curve \(E\), or it can be computed as cofactor \(*\) order.

11. A "prehash" function \(PH\). PureEdDSA means EdDSA where \(PH\) is the identity function, i.e., \(PH(M) = M\). HashEdDSA means EdDSA where \(PH\) generates a short output, no matter how long the message is; for example, \(PH(M) = SHA-512(M)\).

Points on the curve form a group under addition, \((x_3, y_3) = (x_1, y_1) + (x_2, y_2)\), with the formulas
\[
\begin{align*}
x_3 &= \frac{x_1 \cdot y_2 + x_2 \cdot y_1}{1 + d \cdot x_1 \cdot x_2 \cdot y_1 \cdot y_2} \\
y_3 &= \frac{y_1 \cdot y_2 - a \cdot x_1 \cdot x_2}{1 - d \cdot x_1 \cdot x_2 \cdot y_1 \cdot y_2}
\end{align*}
\]
The neutral element in the group is \((0,1)\).

Unlike many other curves used for cryptographic applications, these formulas are "complete"; they are valid for all points on the curve, with no exceptions. In particular, the denominators are non-zero for all input points.

There are more efficient formulas, which are still complete, that use homogeneous coordinates to avoid the expensive modulo \(p\) inversions. See [Faster-ECC] and [Edwards-revisited].

### 3.1. Encoding

An integer \(0 < S < L - 1\) is encoded in little-endian form as a \(b\)-bit string \(ENC(S)\).

An element \((x,y)\) of \(E\) is encoded as a \(b\)-bit string called \(ENC(x,y)\), which is the \((b-1)\)-bit encoding of \(y\) concatenated with one bit that is 1 if \(x\) is negative and 0 if \(x\) is not negative.

The encoding of \(GF(p)\) is used to define "negative" elements of \(GF(p)\): specifically, \(x\) is negative if the \((b-1)\)-bit encoding of \(x\) is lexicographically larger than the \((b-1)\)-bit encoding of \(-x\).

### 3.2. Keys

An EdDSA private key is a \(b\)-bit string \(k\). Let the hash \(H(k) = (h_0, h_1, \ldots, h_{(2b-1)})\) determine an integer \(s\), which is \(2^n\) plus the sum of \(m = 2^i \cdot h_i\) for all integer \(i\), \(c <= i < n\). Let \(s\) determine the multiple \(A = [s]B\). The EdDSA public key is \(ENC(A)\).

The bits \(h_b, \ldots, h_{(2b-1)}\) are used below during signing.
3.3. Sign

The EdDSA signature of a message $M$ under a private key $k$ is defined as the PureEdDSA signature of $\text{PH}(M)$. In other words, EdDSA simply uses PureEdDSA to sign $\text{PH}(M)$.

The PureEdDSA signature of a message $M$ under a private key $k$ is the $2^b$-bit string $\text{ENC}(R) \ || \ \text{ENC}(S)$. $R$ and $S$ are derived as follows. First define $r = H(h_b \ || \ ... \ || \ h_{(2^b-1)} \ || \ M)$ interpreting $2^b$-bit strings in little-endian form as integers in $\{0, 1, ..., 2^{(2^b)} - 1\}$. Let $R = [r]B$ and $S = (r + H(\text{ENC}(R) \ || \ \text{ENC}(A) \ || \ \text{PH}(M)) \times s) \mod L$. The $s$ used here is from the previous section.

3.4. Verify

To verify a PureEdDSA signature $\text{ENC}(R) \ || \ \text{ENC}(S)$ on a message $M$ under a public key $\text{ENC}(A)$, proceed as follows. Parse the inputs so that $A$ and $R$ are elements of $E$, and $S$ is a member of the set $\{0, 1, ..., L-1\}$. Compute $h = H(\text{ENC}(R) \ || \ \text{ENC}(A) \ || \ M)$, and check the group equation $[2^c \times S]B = 2^c \times R + [2^c \times h]A$ in $E$. The signature is rejected if parsing fails (including $S$ being out of range) or if the group equation does not hold.

EdDSA verification for a message $M$ is defined as PureEdDSA verification for $\text{PH}(M)$.

4. PureEdDSA, HashEdDSA, and Naming

One of the parameters of the EdDSA algorithm is the "prehash" function. This may be the identity function, resulting in an algorithm called PureEdDSA, or a collision-resistant hash function such as SHA-512, resulting in an algorithm called HashEdDSA.

Choosing which variant to use depends on which property is deemed to be more important between 1) collision resilience and 2) a single-pass interface for creating signatures. The collision resilience property means EdDSA is secure even if it is feasible to compute collisions for the hash function. The single-pass interface property means that only one pass over the input message is required to create a signature. PureEdDSA requires two passes over the input. Many existing APIs, protocols, and environments assume digital signature algorithms only need one pass over the input and may have API or bandwidth concerns supporting anything else.

Note that single-pass verification is not possible with most uses of signatures, no matter which signature algorithm is chosen. This is because most of the time, one can’t process the message until the signature is validated, which needs a pass on the entire message.
5. EdDSA Instances

This section instantiates the general EdDSA algorithm for the edwards25519 and edwards448 curves, each for the PureEdDSA and HashEdDSA variants (plus a contextualized extension of the Ed25519 scheme). Thus, five different parameter sets are described.

5.1. Ed25519ph, Ed25519ctx, and Ed25519

Ed25519 is EdDSA instantiated with:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>p of edwards25519 in [RFC7748] (i.e., 2^255 - 19)</td>
</tr>
<tr>
<td>b</td>
<td>256</td>
</tr>
<tr>
<td>encoding of GF(p)</td>
<td>255-bit little-endian encoding of {0, 1, ..., p-1}</td>
</tr>
<tr>
<td>H(x)</td>
<td>SHA-512(dom2(phflag,context)</td>
</tr>
<tr>
<td>c</td>
<td>base 2 logarithm of cofactor of edwards25519 in [RFC7748] (i.e., 3)</td>
</tr>
<tr>
<td>n</td>
<td>254</td>
</tr>
<tr>
<td>d</td>
<td>d of edwards25519 in [RFC7748] (i.e., -121665/121666 = 3709570593466943934313808350875465189542113879843219016388785533085940283555)</td>
</tr>
<tr>
<td>a</td>
<td>-1</td>
</tr>
<tr>
<td>B</td>
<td>(X(P),Y(P)) of edwards25519 in [RFC7748] (i.e., (1511222134953540077250115140958531511454012693041857206046113283949847762202, 46316835694926478169428394003475163141307993866256225615783033603165251855960))</td>
</tr>
<tr>
<td>L</td>
<td>order of edwards25519 in [RFC7748] (i.e., 2^252+27742317777372353535851937790883648493).</td>
</tr>
<tr>
<td>PH(x)</td>
<td>x (i.e., the identity function)</td>
</tr>
</tbody>
</table>

Table 1: Parameters of Ed25519

For Ed25519, dom2(f,c) is the empty string. The phflag value is irrelevant. The context (if present at all) MUST be empty. This causes the scheme to be one and the same with the Ed25519 scheme published earlier.

For Ed25519ctx, phflag=0. The context input SHOULD NOT be empty.
For Ed25519ph, phflag=1 and PH is SHA512 instead. That is, the input is hashed using SHA-512 before signing with Ed25519.

Value of context is set by the signer and verifier (maximum of 255 octets; the default is empty string, except for Ed25519, which can’t have context) and has to match octet by octet for verification to be successful.

The curve used is equivalent to Curve25519 [CURVE25519], under a change of coordinates, which means that the difficulty of the discrete logarithm problem is the same as for Curve25519.

5.1.1. Modular Arithmetic

For advice on how to implement arithmetic modulo $p = 2^{255} - 19$ efficiently and securely, see Curve25519 [CURVE25519]. For inversion modulo $p$, it is recommended to use the identity $x^{-1} = x^{(p-2)} \pmod{p}$. Inverting zero should never happen, as it would require invalid input, which would have been detected before, or would be a calculation error.

For point decoding or "decompression", square roots modulo $p$ are needed. They can be computed using the Tonelli-Shanks algorithm or the special case for $p = 5 \pmod{8}$. To find a square root of $a$, first compute the candidate root $x = a^{(p+3)/8} \pmod{p}$. Then there are three cases:

- $x^2 = a \pmod{p}$. Then $x$ is a square root.
- $x^2 = -a \pmod{p}$. Then $2^{(p-1)/4} \cdot x$ is a square root.
- $a$ is not a square modulo $p$.

5.1.2. Encoding

All values are coded as octet strings, and integers are coded using little-endian convention, i.e., a 32-octet string $h h[0]...h[31]$ represents the integer $h[0] + 2^8 \cdot h[1] + ... + 2^{248} \cdot h[31]$.

A curve point $(x,y)$, with coordinates in the range $0 \leq x,y < p$, is coded as follows. First, encode the $y$-coordinate as a little-endian string of 32 octets. The most significant bit of the final octet is always zero. To form the encoding of the point, copy the least significant bit of the $x$-coordinate to the most significant bit of the final octet.
5.1.3. Decoding

Decoding a point, given as a 32-octet string, is a little more complicated.

1. First, interpret the string as an integer in little-endian representation. Bit 255 of this number is the least significant bit of the x-coordinate and denote this value $x_0$. The y-coordinate is recovered simply by clearing this bit. If the resulting value is $\geq p$, decoding fails.

2. To recover the x-coordinate, the curve equation implies $x^2 = (y^2 - 1) / (d y^2 + 1) \pmod{p}$. The denominator is always non-zero mod $p$. Let $u = y^2 - 1$ and $v = d y^2 + 1$. To compute the square root of $(u/v)$, the first step is to compute the candidate root $x = (u/v)^{((p+3)/8)}$. This can be done with the following trick, using a single modular powering for both the inversion of $v$ and the square root:

$$x = (u/v)^{(p+3)/8} = u \cdot v^{-3/2}$$(mod $p$)

3. Again, there are three cases:

1. If $v \cdot x^2 = u \pmod{p}$, $x$ is a square root.
2. If $v \cdot x^2 = -u \pmod{p}$, set $x \leftarrow x \cdot 2^{(p-1)/4}$, which is a square root.
3. Otherwise, no square root exists for modulo $p$, and decoding fails.

4. Finally, use the $x_0$ bit to select the right square root. If $x = 0$, and $x_0 = 1$, decoding fails. Otherwise, if $x_0 \neq x \pmod{2}$, set $x \leftarrow p - x$. Return the decoded point $(x,y)$.

5.1.4. Point Addition

For point addition, the following method is recommended. A point $(x,y)$ is represented in extended homogeneous coordinates $(X, Y, Z, T)$, with $x = X/Z$, $y = Y/Z$, $x \cdot y = T/Z$.

The neutral point is $(0,1)$, or equivalently in extended homogeneous coordinates $(0, Z, Z, 0)$ for any non-zero $Z$. 
The following formulas for adding two points, \((x_3,y_3) = (x_1,y_1) + (x_2,y_2)\), on twisted Edwards curves with \(a=-1\), square \(a\), and non-square \(d\) are described in Section 3.1 of [Edwards-revisited] and in [EFD-TWISTED-ADD]. They are complete, i.e., they work for any pair of valid input points.

\[
A = (Y1-X1)*(Y2-X2) \\
B = (Y1+X1)*(Y2+X2) \\
C = T1*Z1^2*d*T2 \\
D = Z1*Z2 \\
E = B-A \\
F = D-C \\
G = D+C \\
H = B+A \\
X3 = E*F \\
Y3 = G*H \\
T3 = E*H \\
Z3 = F*G
\]

For point doubling, \((x_3,y_3) = (x_1,y_1) + (x_1,y_1)\), one could just substitute equal points in the above (because of completeness, such substitution is valid) and observe that four multiplications turn into squares. However, using the formulas described in Section 3.2 of [Edwards-revisited] and in [EFD-TWISTED-DBL] saves a few smaller operations.

\[
A = X1^2 \\
B = Y1^2 \\
C = 2*Z1^2 \\
H = A+B \\
E = H-(X1+Y1)^2 \\
G = A-B \\
F = C+G \\
X3 = E*F \\
Y3 = G*H \\
T3 = E*H \\
Z3 = F*G
\]
5.1.5. Key Generation

The private key is 32 octets (256 bits, corresponding to b) of cryptographically secure random data. See [RFC4086] for a discussion about randomness.

The 32-byte public key is generated by the following steps.

1. Hash the 32-byte private key using SHA-512, storing the digest in a 64-octet large buffer, denoted h. Only the lower 32 bytes are used for generating the public key.

2. Prune the buffer: The lowest three bits of the first octet are cleared, the highest bit of the last octet is cleared, and the second highest bit of the last octet is set.

3. Interpret the buffer as the little-endian integer, forming a secret scalar s. Perform a fixed-base scalar multiplication [s]B.

4. The public key A is the encoding of the point [s]B. First, encode the y-coordinate (in the range 0 <= y < p) as a little-endian string of 32 octets. The most significant bit of the final octet is always zero. To form the encoding of the point [s]B, copy the least significant bit of the x coordinate to the most significant bit of the final octet. The result is the public key.

5.1.6. Sign

The inputs to the signing procedure is the private key, a 32-octet string, and a message M of arbitrary size. For Ed25519ctx and Ed25519ph, there is additionally a context C of at most 255 octets and a flag F, 0 for Ed25519ctx and 1 for Ed25519ph.

1. Hash the private key, 32 octets, using SHA-512. Let h denote the resulting digest. Construct the secret scalar s from the first half of the digest, and the corresponding public key A, as described in the previous section. Let prefix denote the second half of the hash digest, h[32],...,h[63].

2. Compute SHA-512(dom2(F, C) || prefix || PH(M)), where M is the message to be signed. Interpret the 64-octet digest as a little-endian integer r.

3. Compute the point [r]B. For efficiency, do this by first reducing r modulo L, the group order of B. Let the string R be the encoding of this point.
4. Compute SHA512(dom2(F, C) || R || A || PH(M)), and interpret the 64-octet digest as a little-endian integer k.

5. Compute $S = (r + k \times s) \mod L$. For efficiency, again reduce $k$ modulo $L$ first.

6. Form the signature of the concatenation of $R$ (32 octets) and the little-endian encoding of $S$ (32 octets; the three most significant bits of the final octet are always zero).

5.1.7. Verify

1. To verify a signature on a message $M$ using public key $A$, with $F$ being 0 for Ed25519ctx, 1 for Ed25519ph, and if Ed25519ctx or Ed25519ph is being used, $C$ being the context, first split the signature into two 32-octet halves. Decode the first half as a point $R$, and the second half as an integer $S$, in the range $0 \leq s < L$. Decode the public key $A$ as point $A'$. If any of the decodings fail (including $S$ being out of range), the signature is invalid.

2. Compute SHA512(dom2(F, C) || R || A || PH(M)), and interpret the 64-octet digest as a little-endian integer $k$.


5.2. Ed448ph and Ed448

Ed448 is EdDSA instantiated with:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$p$ of edwards448 in [RFC7748] (i.e., $2^{448} - 2^{224} - 1$)</td>
</tr>
<tr>
<td>$b$</td>
<td>456</td>
</tr>
<tr>
<td>encoding of GF($p$)</td>
<td>455-bit little-endian encoding of ${0, 1, \ldots, p-1}$</td>
</tr>
<tr>
<td>$H(x)$</td>
<td>SHAKE256(dom4(phflag,context)</td>
</tr>
<tr>
<td>$c$</td>
<td>base 2 logarithm of cofactor of edwards448 in [RFC7748] (i.e., 2)</td>
</tr>
<tr>
<td>$n$</td>
<td>447</td>
</tr>
<tr>
<td>$d$</td>
<td>$d$ of edwards448 in [RFC7748] (i.e., -39081)</td>
</tr>
<tr>
<td>$a$</td>
<td>1</td>
</tr>
<tr>
<td>$B$</td>
<td>$(X(P), Y(P))$ of edwards448 in [RFC7748] (i.e., (224580 0402959243001876043409989603624678964163256413424612 54616869504154674060329090291928693579532825780320751 46446173674602635247710, 2988192100784814926760179304 43930673437544040154080242095928241372331506189835876 00353687865541878473398230323350346250053154506283266 0))</td>
</tr>
<tr>
<td>$L$</td>
<td>order of edwards448 in [RFC7748] (i.e., $2^{446} - 13818$ (06680989511535200738674851542688033669247488217860989 4547503885).)</td>
</tr>
<tr>
<td>$PH(x)$</td>
<td>$x$ (i.e., the identity function)</td>
</tr>
</tbody>
</table>

Table 2: Parameters of Ed448

Ed448ph is the same but with $PH$ being SHAKE256($x$, 64) and phflag being 1, i.e., the input is hashed before signing with Ed448 with a hash constant modified.

Value of context is set by signer and verifier (maximum of 255 octets; the default is empty string) and has to match octet by octet for verification to be successful.

The curve is equivalent to Ed448-Goldilocks under change of the basepoint, which preserves difficulty of the discrete logarithm.
5.2.1. Modular Arithmetic

For advice on how to implement arithmetic modulo \( p = 2^{448} - 2^{224} - 1 \) efficiently and securely, see [ED448]. For inversion modulo \( p \), it is recommended to use the identity \( x^{-1} = x^{(p-2)} \mod p \). Inverting zero should never happen, as it would require invalid input, which would have been detected before, or would be a calculation error.

For point decoding or "decompression", square roots modulo \( p \) are needed. They can be computed by first computing candidate root \( x = a^{(p+1)/4} \mod p \) and then checking if \( x^2 = a \). If it is, then \( x \) is the square root of \( a \); if it isn’t, then \( a \) does not have a square root.

5.2.2. Encoding

All values are coded as octet strings, and integers are coded using little-endian convention, i.e., a 57-octet string \( h_0,\ldots,h_{56} \) represents the integer \( h_0 + 2^8 \cdot h_1 + \ldots + 2^{448} \cdot h_{56} \).

A curve point \((x,y)\), with coordinates in the range \( 0 \leq x,y < p \), is coded as follows. First, encode the \( y \)-coordinate as a little-endian string of 57 octets. The final octet is always zero. To form the encoding of the point, copy the least significant bit of the \( x \)-coordinate to the most significant bit of the final octet.

5.2.3. Decoding

Decoding a point, given as a 57-octet string, is a little more complicated.

1. First, interpret the string as an integer in little-endian representation. Bit 455 of this number is the least significant bit of the \( x \)-coordinate, and denote this value \( x_0 \). The \( y \)-coordinate is recovered simply by clearing this bit. If the resulting value is \( \geq p \), decoding fails.

2. To recover the \( x \)-coordinate, the curve equation implies \( x^2 = (y^2 - 1) / (d \cdot y^2 - 1) \mod p \). The denominator is always non-zero mod \( p \). Let \( u = y^2 - 1 \) and \( v = d \cdot y^2 - 1 \). To compute the square root of \((u/v)\), the first step is to compute the candidate root \( x = (u/v)^{(p+1)/4} \). This can be done using the following trick, to use a single modular powering for both the inversion of \( v \) and the square root:

\[
\begin{align*}
(p+1)/4 & \quad 3 & \quad (p-3)/4 \\
x = (u/v) & = u \cdot v^{(u^5 \cdot v^3)} & \mod p
\end{align*}
\]
3. If \( v \cdot x^2 = u \), the recovered \( x \)-coordinate is \( x \). Otherwise, no square root exists, and the decoding fails.

4. Finally, use the \( x_0 \) bit to select the right square root. If \( x = 0 \), and \( x_0 = 1 \), decoding fails. Otherwise, if \( x_0 \neq x \mod 2 \), set \( x \leftarrow p - x \). Return the decoded point \((x, y)\).

5.2.4. Point Addition

For point addition, the following method is recommended. A point \((x, y)\) is represented in projective coordinates \((X, Y, Z)\), with \( x = X/Z \), \( y = Y/Z \).

The neutral point is \((0,1)\), or equivalently in projective coordinates \((0, Z, Z)\) for any non-zero \( Z \).

The following formulas for adding two points, \((x_3, y_3) = (x_1, y_1) + (x_2, y_2)\) on untwisted Edwards curve (i.e., \( a=1 \)) with non-square \( d \), are described in Section 4 of [Faster-ECC] and in [EFD-ADD]. They are complete, i.e., they work for any pair of valid input points.

\[
A = Z_1 \cdot Z_2 \\
B = A^2 \\
C = X_1 \cdot X_2 \\
D = Y_1 \cdot Y_2 \\
E = d \cdot C \cdot D \\
F = B - E \\
G = B + E \\
H = (X_1 + Y_1) \cdot (X_2 + Y_2) \\
X_3 = A \cdot F \cdot (H - C - D) \\
Y_3 = A \cdot G \cdot (D - C) \\
Z_3 = F \cdot G
\]
Again, similar to the other curve, doubling formulas can be obtained by substituting equal points, turning four multiplications into squares. However, this is not even nearly optimal; the following formulas described in Section 4 of [Faster-ECC] and in [EFD-DBL] save multiple multiplications.

\[
\begin{align*}
B &= (X1+Y1)^2 \\
C &= X1^2 \\
D &= Y1^2 \\
E &= C+D \\
H &= Z1^2 \\
J &= E-2*H \\
X3 &= (B-E)*J \\
Y3 &= E*(C-D) \\
Z3 &= E*J
\end{align*}
\]

5.2.5. Key Generation

The private key is 57 octets (456 bits, corresponding to b) of cryptographically secure random data. See [RFC4086] for a discussion about randomness.

The 57-byte public key is generated by the following steps:

1. Hash the 57-byte private key using SHAKE256(x, 114), storing the digest in a 114-octet large buffer, denoted h. Only the lower 57 bytes are used for generating the public key.

2. Prune the buffer: The two least significant bits of the first octet are cleared, all eight bits the last octet are cleared, and the highest bit of the second to last octet is set.

3. Interpret the buffer as the little-endian integer, forming a secret scalar s. Perform a known-base-point scalar multiplication [s]B.

4. The public key A is the encoding of the point [s]B. First encode the y-coordinate (in the range 0 <= y < p) as a little-endian string of 57 octets. The most significant bit of the final octet is always zero. To form the encoding of the point [s]B, copy the least significant bit of the x coordinate to the most significant bit of the final octet. The result is the public key.
5.2.6. Sign

The inputs to the signing procedure is the private key, a 57-octet string, a flag $F$, which is 0 for Ed448, 1 for Ed448ph, context $C$ of at most 255 octets, and a message $M$ of arbitrary size.

1. Hash the private key, 57 octets, using SHAKE256($x$, 114). Let $h$ denote the resulting digest. Construct the secret scalar $s$ from the first half of the digest, and the corresponding public key $A$, as described in the previous section. Let prefix denote the second half of the hash digest, $h[57], ..., h[113]$.

2. Compute SHAKE256(dom4($F$, $C$) || prefix || PH($M$), 114), where $M$ is the message to be signed, $F$ is 1 for Ed448ph, 0 for Ed448, and $C$ is the context to use. Interpret the 114-octet digest as a little-endian integer $r$.

3. Compute the point $[r]B$. For efficiency, do this by first reducing $r$ modulo $L$, the group order of $B$. Let the string $R$ be the encoding of this point.

4. Compute SHAKE256(dom4($F$, $C$) || $R$ || $A$ || PH($M$), 114), and interpret the 114-octet digest as a little-endian integer $k$.

5. Compute $S = (r + k * s) \mod L$. For efficiency, again reduce $k$ modulo $L$ first.

6. Form the signature of the concatenation of $R$ (57 octets) and the little-endian encoding of $S$ (57 octets; the ten most significant bits of the final octets are always zero).

5.2.7. Verify

1. To verify a signature on a message $M$ using context $C$ and public key $A$, with $F$ being 0 for Ed448 and 1 for Ed448ph, first split the signature into two 57-octet halves. Decode the first half as a point $R$, and the second half as an integer $S$, in the range $0 \leq s < L$. Decode the public key $A$ as point $A'$. If any of the decodings fail (including $S$ being out of range), the signature is invalid.

2. Compute SHAKE256(dom4($F$, $C$) || $R$ || $A$ || PH($M$), 114), and interpret the 114-octet digest as a little-endian integer $k$.


6. Ed25519 Python Illustration

The rest of this section describes how Ed25519 can be implemented in Python (version 3.2 or later) for illustration. See Appendix A for the complete implementation and Appendix B for a test-driver to run it through some test vectors.

Note that this code is not intended for production as it is not proven to be correct for all inputs, nor does it protect against side-channel attacks. The purpose is to illustrate the algorithm to help implementers with their own implementation.

## First, some preliminaries that will be needed.

```python
import hashlib

def sha512(s):
    return hashlib.sha512(s).digest()

# Base field Z_p
p = 2**255 - 19

def modp_inv(x):
    return pow(x, p-2, p)

# Curve constant
d = -121665 * modp_inv(121666) % p

# Group order
q = 2**252 + 2774231777372353585193790883648493

def sha512_modq(s):
    return int.from_bytes(sha512(s), "little") % q

## Then follows functions to perform point operations.

# Points are represented as tuples (X, Y, Z, T) of extended
# coordinates, with x = X/Z, y = Y/Z, x*y = T/Z

def point_add(P, Q):
    A, B = (P[1]-P[0]) * (Q[1]-Q[0]) % p, (P[1]+P[0]) * (Q[1]+Q[0]) % p;
    E, F, G, H = B-A, D-C, D+C, B+A;
    return (E*F, G*H, F*G, E*H);
```
# Computes \( Q = s \cdot P \)

```python
def point_mul(s, P):
    Q = (0, 1, 1, 0)  # Neutral element
    while s > 0:
        if s & 1:
            Q = point_add(Q, P)
        P = point_add(P, P)
        s >>= 1
    return Q
```

```python
def point_equal(P, Q):
    # \( x_1 / z_1 = x_2 / z_2 \iff x_1 \cdot z_2 = x_2 \cdot z_1 \)
    if (P[0] * Q[2] - Q[0] * P[2]) % p != 0:
        return False
        return False
    return True
```

## Now follows functions for point compression.

```python
# Square root of -1
modp_sqrt_m1 = pow(2, (p-1) // 4, p)

# Compute corresponding x-coordinate, with low bit corresponding to
# sign, or return None on failure

def recover_x(y, sign):
    if y >= p:
        return None
    x2 = (y*y-1) * modp_inv(d*y*y+1)
    if x2 == 0:
        if sign:
            return None
        else:
            return 0
    # Compute square root of x2
    x = pow(x2, (p+3) // 8, p)
    if (x*x - x2) % p != 0:
        x = x * modp_sqrt_m1 % p
    if (x*x - x2) % p != 0:
        return None
    if (x & 1) != sign:
        x = p - x
    return x
```
# Base point

g_y = 4 * modp_inv(5) % p
g_x = recover_x(g_y, 0)
G = (g_x, g_y, 1, g_x * g_y % p)

def point_compress(P):
    zinv = modp_inv(P[2])
    x = P[0] * zinv % p
    y = P[1] * zinv % p
    return int.to_bytes(y | ((x & 1) << 255), 32, "little")

def point_decompress(s):
    if len(s) != 32:
        raise Exception("Invalid input length for decompression")
    y = int.from_bytes(s, "little")
    sign = y >> 255
    y &= (1 << 255) - 1
    x = recover_x(y, sign)
    if x is None:
        return None
    else:
        return (x, y, 1, x*y % p)

## These are functions for manipulating the private key.

def secret_expand(secret):
    if len(secret) != 32:
        raise Exception("Bad size of private key")
    h = sha512(secret)
    a = int.from_bytes(h[:32], "little")
    a &= (1 << 254) - 8
    a |= (1 << 254)
    return (a, h[32:])

def secret_to_public(secret):
    (a, dummy) = secret_expand(secret)
    return point_compress(point_mul(a, G))
def sign(secret, msg):
    a, prefix = secret_expand(secret)
    A = point_compress(point_mul(a, G))
    r = sha512_modq(prefix + msg)
    R = point_mul(r, G)
    Rs = point_compress(R)
    h = sha512_modq(Rs + A + msg)
    s = (r + h * a) % q
    return Rs + int.to_bytes(s, 32, "little")

def verify(public, msg, signature):
    if len(public) != 32:
        raise Exception("Bad public key length")
    if len(signature) != 64:
        Exception("Bad signature length")
    A = point_decompress(public)
    if not A:
        return False
    Rs = signature[:32]
    R = point_decompress(Rs)
    if not R:
        return False
    s = int.from_bytes(signature[32:], "little")
    if s >= q: return False
    h = sha512_modq(Rs + public + msg)
    sB = point_mul(s, G)
    hA = point_mul(h, A)
    return point_equal(sB, point_add(R, hA))

7. Test Vectors

This section contains test vectors for Ed25519ph, Ed25519ctx,
Ed448ph, Ed25519, and Ed448.

Each section contains a sequence of test vectors. The octets are hex
encoded, and whitespace is inserted for readability. Ed25519,
Ed25519ctx, and Ed25519ph private and public keys are 32 octets;
signatures are 64 octets. Ed448 and Ed448ph private and public keys
are 57 octets; signatures are 114 octets. Messages are of arbitrary
length. If the context is non-empty, it is given as 1-255 octets.
7.1. Test Vectors for Ed25519

These test vectors are taken from [ED25519-TEST-VECTORS] (but we removed the public key as a suffix of the private key and removed the message from the signature) and [ED25519-LIBGCRYPT-TEST-VECTORS].

-----TEST 1

ALGORITHM: Ed25519

SECRET KEY:
9d61b19defffd5a60ba844af492ec2cc4
4449c5697b326919703bac031caef6f0

PUBLIC KEY:
d75a980182b10ab7d54bfed3c964073a
0ee172f3daa62325af021a68f707511a

MESSAGE (length 0 bytes):

SIGNATURE:
e5564300c360ac729086e2cc806e828a
84877f1eb8e5d974d873e06522490155
5fb8821590a33bacc61e39701cf9b46b
d25bf5f0595bb82465514138e7a100b

-----TEST 2

ALGORITHM: Ed25519

SECRET KEY:
4ccd089b28ff96da9db6c346ec114e0f
5b8a319f35aba624da8cf6ed4fb8a6fb

PUBLIC KEY:
3d4017c3e848395a92b70aa74d1b7ebc
9c982ccf2ec4968cc0cd55f12af4660c

MESSAGE (length 1 byte):
72

SIGNATURE:
92a009a9f0d4cab8720e820b5f642540
a2b27b5416503f8fb3762223ebdb69da
085acle43e15996e458f3613d0f11d8c
387b2aeb4302aebe00d291612bb0c00
-----TEST 3

ALGORITHM:
Ed25519

SECRET KEY:
c5aa8df43f9f837bedb7442f31dcb7b1
66d38535076f094b85ce3a2e0b4458f7

PUBLIC KEY:
fc51cd8e6218a1a38da47ed00230f058
0816ed13ba3303ac5deb911548908025

MESSAGE (length 2 bytes):
af82

SIGNATURE:
6291d657deec240242872e69c3abe01a3
0ce548a284743a445e3680d7db5ac3ac
18ff9b538d16f290ae67f760984dc659
4a7c15e9716ed28dc027beceeealec40a

-----TEST 1024

ALGORITHM:
Ed25519

SECRET KEY:
f5e5767cf153319517630f226876b86c
8160cc583bc013744c6bf255f5cc0ee5

PUBLIC KEY:
278117fc144c72340f67d0f2316e8386
ceffbf2b2428c9c51fef7c597f1d426e

MESSAGE (length 1023 bytes):
08b8b2b733424243760fe426a4b54908
632110a66c2f6591eabd3345e3e4eb98
fa6e264bf09efe12ee50f8f54ef9f77b1
e355f6c50544e23fb1433ddf73be84d8
79de7c0046dc4996d9e773f4bc9efe57
38829adb2681b37c93a1b270b20329d
658675fc6ea534e0810a4432826bf58c
941efb65d57a338bbd2e26640f89ffbc
1a858efcb8550ee3a5e1998bd177e93a
7363c344fe6b199ee5d02e82d522c4fe
ba15452f80288a821a579116ec6dad2b
3b310da903401aa62100ab5d1a36553e
06203b33890cc9b832f79ef805600cc9
a39ce767967ed628ce6ad573cb116dbef
efd75499da6bd68a8a7b92928a8bcb10
3b6621fcede2beaca1231d206be6cd9e9c7
aff6f6c94fcd7204ed3455c68c3f4a4
1da4af2b74ef5c5f1d8ac70bdc87ed1
85ce81bd45395d44254d95629e9855a9
4a7c1958d18ada5d0532ed8a5aa3fb2
d17ba70eb6248e594e1a2297acbb39d
502f1a8c6eb6f1ce223bd1a1af40cc24
554119a831a9aad6079cad88425de6bd
e1a9187eb6092cf67bf2b13fd65f270
88d78b7e883c8759d2c4f5c65adb7553
878ad57f59fad878e80a0c9ba63bc4ccc
2732e69485b9c90bfb62481d9089b
ecff80ce2df162a4df5b92dd597b07
07e0917af4b6ff75fed413d238f555a
7a569d80c3414a6d0859d65a46128ba
b27af17a1314f318c7c26b23e6e080b
82b0ce26401d2e22f04d831255cd51a
dddd3b752a1ae0784504df543af8969b
ea3ea708ff7c9888c44da2af58429e
c96031dbcad3dad9af0dcbaabaf268cb8
fcffead94f3c7ca495e056a9b47acb7b
51f73e866c6655ade8297297d07a1d1
ba5e43f1bca3230165139e22904cc8c
42f58c30c04aafdb038dda0847dd988d
cda6f3bf1d54b4c4525004aa6e0eff8
casa61783aaaccc57f3d1f92b0fe2fd1a8
5f6725417b6e614d68086f66e34df
f7310fdec8a2ebfd904b0e1dc5ab2927
094b2db68d6f903b68401adebf5a7e08
78ff4ef5d6365a65040cf9bf4acca7
984a74d37145988708fc0b16ac451649
d61887adbdd191f64b5f5e2ab47b57
f7f7276cd419c17a3ca8e1b939ae49e4
88acaba695610b5480109c8b17b380e1
b7b750dfc7598d5d011fd2ddcc5600a3
2ef5b52a1e3cc3850e306aa342721aacc09
43b6f68664b2579376504cc4939d97e
6ad3f3fb0f9cd71a43dd497f01f17c0e2
cc3f797aa2a2f25665616e6c496afc5f
b93246f6b1116398a346f1a641f3b041
e989f7914f90cc27ff357876e506b5
0d334ba77c225bc307ba537152f3f161
0e4eafe595f6d9d9d11faa933a15eef1
369546868a7f3a45a96768d40fd9d034
12091c6315cf4fde7c8b609693780d
SIGNATURE:
0aab4c900501b3e24d7cddf4663326a3a
87df5e4843b2cbdb67cbafe460fec350
aa5371b1508f9f4528ecea23c436d94b
5e8fcd4f681e30a6ac00a9704a188a03
-----TEST SHA(abc)

ALGORITHM:
Ed25519

SECRET KEY:
833fe62409237b9d62ec77587520911e
9a759ce1d91755b7da901b96dca3d42

PUBLIC KEY:
ce172b93ad5e563bf4932c70e1245034
c35467ef2efd4d64ebf819683467e2bf

MESSAGE (length 64 bytes):
ddaf35a193617abacc417349ae204131
12e6fa4e89a97ea20a9e14ee64b55d39a
2192992a274fc1a836ba3c23a3fee8bd
454d4423643ce80e2a9ac94fa54ca49f

SIGNATURE:
dc2a4459e7369633a52b1bf277789a00
201009a3efbf3ecb69bea2186c26b589
09351fc9ac903ecdfb7c66431e030
3dca179c138ac17ad9bef1177331a704
-----

7.2. Test Vectors for Ed25519ctx

-----foo

ALGORITHM:
Ed25519ctx

SECRET KEY:
0305334e381af78f141cb666f6199f57
bc3495335a256a95bd2a55bf546663f6
PUBLIC KEY:
dfc9425e4f968f7f0c29f0259cf5f9ae
d6851c2bb4ad8bbf860cfee0ab248292

MESSAGE (length 16 bytes):
f726936d19c800494e3fdaff20b276a8

CONTEXT:
666f6f

SIGNATURE:
55a4cc2f70a54e04288c5f4cd1e45a7b
b520b36292911876cada7323198dd87a
8b36950b9513022907a7fb7c4e9b2d5
f6cca685a587b4b21f4b888e4e7edb0d

-----bar

ALGORITHM:
Ed25519ctx

SECRET KEY:
0305334e381af78f141cb666f6199f57
bc3495335a256a95bd2a55bf546663f6

PUBLIC KEY:
dfc9425e4f968f7f0c29f0259cf5f9ae
d6851c2bb4ad8bbf860cfee0ab248292

MESSAGE (length 16 bytes):
f726936d19c800494e3fdaff20b276a8

CONTEXT:
626172

SIGNATURE:
fc60d5872fc46b3aa69f8b5b4351d580
8f92bcc044606db097abab6dbcb1ae3
216c48e8b3b6643b5b186d1d28f8ee1
5a5ca2df6668346291c2043d4eb3e90d

-----foo2

ALGORITHM:
Ed25519ctx
SECRET KEY:
0305334e381af78f141cb666f6199f57
bc3495335a256a95bd2a55bf546663f6

PUBLIC KEY:
dfc9425e4f968f7f0c29f0259cf5f9ae
d6851c2bb4ad8bfb860cf0e0ab248292

MESSAGE (length 16 bytes):
508e9e6882b979fe900f62adceaca35

CONTEXT:
666f6f

SIGNATURE:
8b70c1cc8310e1de20ac53ce28ae6e72
07f33c3295e03bb5c0732a1d20dc6490
8922a8b052cf99b7c4fe107a5abb5b2c
4085ae75890d02df26269d8945f84b0b

-----foo3

ALGORITHM:
Ed25519ctx

SECRET KEY:
ab9c2853ce297ddab85c993b3ae14bca
d39b2c682beabc27d6d4eb20711d6560

PUBLIC KEY:
0f1d1274943b91415889152e893d80e9
3275a1fc0b65fd71b4b0dda10ad7d772

MESSAGE (length 16 bytes):
f726936d19c800494e3fdaa20b276a8

CONTEXT:
666f6f

SIGNATURE:
21655b5f1aa965996b3f97b3c849eafb
a922a0a62992f73b3d1b73106a84ad85
e9b86a7b6005ea868337ff2d20a7f5fb
d4cd10b0be49a68da2b2e0dc0ad8960f

------
7.3. Test Vectors for Ed25519ph

-----TEST abc

ALGORITHM: Ed25519ph

SECRET KEY:
833fe62409237b9d62ec77587520911e
9a759ce1d19755b7da901b96dca3d42

PUBLIC KEY:
ec172b93ad56e563bf4932c70e1245034
c35467ef2efd4d64ebf819683467e2bf

MESSAGE (length 3 bytes):
616263

SIGNATURE (length 3 bytes):
98a70222f0b8121aa9d30f813d683f80
9e462b469c7ff8763949bb9ae6dae41
31f85042463c2a355a2003d062ad5aa
a10b8c61e636062aaad11c2a26083406

7.4. Test Vectors for Ed448

-----Blank

ALGORITHM: Ed448

SECRET KEY:
6c82a562cb808d10d632be89c8513ebf
6c929f34ddfa8c9f63c9960ef6e348a3
528c8a3fc2f044e39a3fc5b94492f8f
032e7549a20098f95b

PUBLIC KEY:
5fd7449b59b461fd2ce787ec616ad46a
1da1342485a70e1f8a0ea75d80e96778
edf124769b46c7061bd6783df1e50f6c
d1fa1abeafe8256180

MESSAGE (length 0 bytes):
SIGNATURE:
533a37f6bbe457251f023c0d88f976ae
2dfb504a843e34d2074fd823d41a591f
2b233f034f628281f2fd7a22ddd47d78
28c59bd0a21bdf3980ff0d2028d4b18a
9df63e006c5d1c2d345b925d8dc00b41
04852db99ac5c7cd8a8530a113a04fb
b61149f05a7363268c71d95808ff2e65
2600

-----1 octet

ALGORITHM:
Ed448

SECRET KEY:
c4eab05d357007c632f3dbb48489924d
552b08fe0c353a0d4a1f00acda2c463a
fbea67c5e8d2877c5e3bc397a659949e
f802e954e0a12274e

PUBLIC KEY:
43ba28f430cddf456ae531545f7ecd0a
c834a55d9358c0372bfa0c6c6798c086
6aea01eb00742802b8438ea4cb82169c
235160627b4c3a9480

MESSAGE (length 1 byte):
03

SIGNATURE:
26b8f91727bd62897af15e41eb43c377
ebf9c610d48f2335cb0bd0087810f435
2541b143c4b981b7e18f62de8ccdf633
fcb1f037ab7cd779805e0dbcc0aae1cb
ce1af2be027df36bc04dcecbf154336
c19f0af7e0a6472905e799f19532a0f
f3348ab21aa4adafdf1d234441cf807c0
3a00

-----1 octet (with context)

ALGORITHM:
Ed448
SECRET KEY:
c4eab05d357007c632f3dbb48489924d
552b08fe0c353a0d4a1f00acda2c463a
fbea67c5e8d2877c5e3bc397a659949e
f8021e954e0a12274e

PUBLIC KEY:
43ba28f430cddf456ae531545f7ecd0a
c834a55d9358c0372bfa0c6c6798c086
6aea01eb00742802b8438ea4cb82169c
235160627b4c3a9480

MESSAGE (length 1 byte):
03

CONTEXT:
666f66f

SIGNATURE:
d4f8f6131770dd46f40867d6fd5d5055
dec43541f8c5e35abbc0d01b32a89f7d2
151f7647f1d8ca2ae279fb842d60721
7f96e042f6815ea000c85741de5c8da
1144a6a1aba7f96de42505d7a7298524
fda538fcccbb754f578c1cad10d54d0d
5428407e85dc9c98a49155c13764e66c
3c00

-----11 octets

ALGORITHM:
Ed448

SECRET KEY:
cd23d24f714274e744343237b93290f5
11f6425f98e64459ff203e8985083fffd
f60500553abc0e05cd02184b8b89c4cc
d67e187951267eb328

PUBLIC KEY:
dcea9e78f35a1bf3499a831b10b86c90
acc01cd84b67a0109b55a36e9328b1e3
65f9e161d71ce7131a543ea4cb5f7e9f
1d8b00696447001400

MESSAGE (length 11 bytes):
0c3e544074ec63b0265e0c
SIGNATURE:
1f0a888ce25e8d458a21130879b840a
9089d99aaba039eaf3e3afa090a09d3
89dba82c4ff2ae8ac5cdfe7c55e94d5d
961a29fe0109941e00b8ddbeea6d3b05
1068df7254c0c0d1c29cbe62db2dc957d
bb47b51fd3f213fb8698f064774250a5
028961c9bf8f8f8f973fe5d5c206492b14
0e00

-----12 octets
ALGORITHM:
Ed448
SECRET KEY:
258cdd4ada32ed9c9ff54e63756ae582
fb8fab2ac721f2c8e676a72768513d93
9f63dddb55609133f29af86ec9929dc
ch52c1c5fd2ff7e21b
PUBLIC KEY:
3ba16da0c6f2cc1f30187740756f5e79
8d6bc5fc015d7c63cc9510ee3fd4adc
24d8e96b6e64f94d19b945361726b
d75e149ef09817f580
MESSAGE (length 12 bytes):
64a65f3cdedcdd6811e2915
SIGNATURE:
7eeeab7c4e50fb799b418ee5e3197ff6
bf15d43a14c34389b59d1a7b1b85b4a
e90438aca634bea45e3a2695f1270f07
fdccdf7c62b8eefaf00b45c2c96ba457e
b1a8bf075a3db28e524f6b923ed4ad7
47c3c9e03c7079efb87cb110d3a99861
e72003c2ae6db8b827e4e6c143064ff
3c00

-----13 octets
ALGORITHM:
Ed448
SECRET KEY:
7ef4e84544236752fbb56b8f31a23a10
e42814f5f55ca037cddc11c64c9a3b29
49c1bb60700314611732a6c2f6a98eed
c0266a11a93970100e

PUBLIC KEY:
b3da079b0aa493a5772029f0467baede
5ae81122d93a22532361da294f7bb381
5c5dc59e176b4d9f381ca0938e13c6c0
7b174be65dfa578e80

MESSAGE (length 13 bytes):
64a65f3cdedcd6d6811e2915e7

SIGNATURE:
6a12066f55331b6c22acd5b5f5d712
28fbd8a0ae8dec26bdd306743c5027cb
4890810c162027468675ecf645a8317
6cd7323a2ccde2d80efe5a1268e8aca
1d6fbc194d3f77c44986eb4ab4177919
ad8bec3b47bb5fc6e28196fd1caf5
6b4e7e0ba519234d047155ac727a105
3100

-----64 octets

ALGORITHM:
Ed448

SECRET KEY:
d65df341ad13e008567688baedd8a8e9d
cdc17dc024974ea5b4227b6530e339bf
f21f99e68ca6968f3ccca6defe0fb9f4fa
b4fa135d5542ea3f01

PUBLIC KEY:
df9705f58edbab802c7f8363cfe5560a
b1c6132c20a9f1dd163483a26f8ac5a
39d680bf41dfbd261b099bb03b3fb5
0906cb28bd8a081f00

MESSAGE (length 64 bytes):
bd0f6a20747d561bddd4640a332461a
4a30a12a434cd0bf40d766d9c6d458e5
512204a30c17df50b5079631f64eb31
12182da300583546113718d1a5ef944
SIGNATURE:
554bc2480860b49eab8532d2a533b7d5
78ef473eeb58c98bb2d0e1ce488a98b1
8dfde9b9b9077567f47d4a1c3482058
efc9f40d2ca033a801b63d45b3b722e
f552bad3b4cc667da35019261c508c
f7b6b5adadc2c8d9a446ef003fb05cba
5f30e88e36ec2703b349ca229c267083
3900

-------256 octets

ALGORITHM:
Ed448

SECRET KEY:
2ec5fe3c17045abdb136a5e6a913e32a
b75ae88b53d2fc149b77e504132d3756
9b7e766ba74a19bd6162343a21c8590a
a9cebca9014c636df5

PUBLIC KEY:
79756f014dcfe2079f5dd9e718be4171
e2ef2486a08f25186f6bff43a9936b9b
fe12402b08a65798a3d81e22e9ec80e
7690862ef3d4ed3a00

MESSAGE (length 256 bytes):
15777532b0bdd0d1389f636c5f6b9ba7
34c90af572877e2d272dd078aa1e567c
fa80e12928bb542330e8409f31745041
07ecd5efac61ae7504dabe2a602ede89
e5cca6257a7c77e27a702b3ae39fc769
fc54f2395ae6a1178cab4738e543072f
c1c177fe71e92e25bf03e4eb72f47b6
4d0465aae4c7fad372536c8ba516a60
39c3c2a39f0e4d832be432df9ea706a6
e5c7e19f397964ca42580027c0541b5
90316dcb5622b6b2a6fe7a4abfffd9610
5eca76ea7b98816af0748c10df048ce0
12d901015a51f189f3888145c03650a
23ce894c3bd889e030d565071c59f409
a9981b51878fd6fc110624dcbcede0bf7
a69ccce38fabdf86f3bef6044819de11
SIGNATURE:
c650d6bb0601c19ca11439e1640dd931
f43c518ea5bea70d3dcde5f4191fe53f
00cf96654672b2c7d58be2b9badef28
743954e3a4a23f880e8d4f1cfc6e2d7a
61452d26da05896f0a50da66a239a8a1
88b6d825b3305ad77b73f9b0836ecc6
0987fd08527c1a8e08d5823e65cafe2a
3d00

-----1023 octets

ALGORITHM:
Ed448

SECRET KEY:
872d093780f5d3730da382cd4af377634ec
44dc54f1c2ed9bea86fafa7632d8be19
9ea165f5ad55dd9ce8

PUBLIC KEY:
a81b2e8a70a5ac94ffdbcc9badfc3feb
0801f25857bb114ad44ece1ec0e799d
a0eefb81cd685c056f64ee6f8c61c3873738cf400

MESSAGE (length 1023 bytes):
6ddf802e1eae4986935f7f981ba3f035
1d6273c0a0cc22c9c0e8339168e675412
a3debf8af435ed651558007db4384b650
fcc07e3b586a27a4f7a00ac8a66f6c2cd
86ae4bf1570c4e6a40c931db27b2faa
15a8cdd52c7f7362c46e23dae0fbc
3a79b6806e316ef7c7b6819bf66bc76
a26067a53f396adafdbdc11c77f7777e9
72660cf4b69b369a665f02e0cc9b6e
dfad136b4fa7e723d2813db3136dffe9
b6d044322fee2947952e03173ab5c60
3349b307bcd27bc6cb8b8bd7bd32231
9b8033a581b59eadebb09b34f3d2277
d4f0343624acc817804728b25ab79717
2b4c5c21a2f9c7839d64300232eb66e
53f31c723fa37fe387cd3e50bdf9813
a30e5bb12cf4cd930c40cfb4e1cf6225
92a49588794494d56d24ea4b040c89fc0
547cc9e9b61c8cb10adde976a5d602b
1cf85b9b9a00led3c6a4d3b1437f520
SIGNATURE:
e301345a41a39a4d72fff8df69c98075
a0cc082b802fc9b2b6bc503f926b65bd
df7f4c8f1cb49f6396afc8a70abe6d8a
ef0db478d4c6b2970076c6a0484fe76d
76b3a97625d79f1ce240e7c576750d29
5528286f719b413de9ada3e8eb78ed57
3603ce30d8bb761785dc30dbc320869e
1a00
-----

7.5. Test Vectors for Ed448ph

-----TEST abc

ALGORITHM:
Ed448ph

SECRET KEY:
833fe62409237b9d62ec77587520911e
9a759cec1d19755b7da901b96dca3d42
ef722e0d51041d205d6dbefde69e3
ab22ec7867c6e2c49

PUBLIC KEY:
259b71c19f83ef777a7abd26524cbdb31
61b590a48f7d17de3ee0ba9c52beb743
c09428a131d6b157303d90d8132c276
d5ed3d5d01c0f53880

MESSAGE (length 3 bytes):
616263

SIGNATURE:
822f6901f7480f3d5f562c592994d969
3602875614483256505600bcb281ae38
1f54d6bce2ea911574932f52a4e6cadd
78769375ec3ff1b801a0d9b3f4030cd
433964a6457ea394765112f4f97469b5
7dd32dbc560a9a94d00b0f07620464a3
ad203df7dc7ce360c3cd3696d9d9fab9
0f00
-----TEST abc (with context)

ALGORITHM:
Ed448ph

SECRET KEY:
833fe62409237b9d62ec77587520911e
9a759cec1d19755b7da901b96dca3d42
ef7822e0d5104127dc05d6dbe6e69e3
ab2ceca7c867c6e2c49

PUBLIC KEY:
259b71c19f83ef77a7abd26524cbdb31
61b590a48f7d17de3ee0ba9c52beb743
c09428a131d6b1b57303d90d8132c276
d5ed3d5d01c0f53880

MESSAGE (length 3 bytes):
616263

CONTEXT:
666f6f6

SIGNATURE:
c32299d46ec8ff02b54540982814dce9
a05812f81962b649d528095916a2aa48
1065b150423ef927ecf0af5888f90da
0f6a9a85ad5dc3f280d91224ba9911a3
653d00e484e2ce232521481c8658df30
4bb7745a73514cdb9bf3e15784ab7128
4f8d0704a608c54a6b62d97beb511d13
2100

-----
8. Security Considerations

8.1. Side-Channel Leaks

For implementations performing signatures, secrecy of the private key is fundamental. It is possible to protect against some side-channel attacks by ensuring that the implementation executes exactly the same sequence of instructions and performs exactly the same memory accesses, for any value of the private key.

To make an implementation side-channel silent in this way, the modulo p arithmetic must not use any data-dependent branches, e.g., related to carry propagation. Side-channel silent point addition is straightforward, thanks to the unified formulas.

Scalar multiplication, multiplying a point by an integer, needs some additional effort to implement in a side-channel silent manner. One simple approach is to implement a side-channel silent conditional assignment, and use it together with the binary algorithm to examine one bit of the integer at a time.

Compared to other signature schemes, avoiding data-dependent branches is easier due to side-channel silent modulo p arithmetic being easier (with recommended curves) and having complete addition formulas instead of having a number of special cases.

Note that the example implementations in this document do not attempt to be side-channel silent.

8.2. Randomness Considerations

EdDSA signatures are deterministic. This protects against attacks arising from signing with bad randomness; the effects of which can, depending on the algorithm, range up to full private key compromise. It can be surprisingly hard to ensure good-quality random numbers, and there have been numerous security failures relating to this.

Obviously, private key generation requires randomness, but due to the fact that the private key is hashed before use, a few missing bits of entropy doesn’t constitute a disaster.

The basic signature verification is also deterministic. However, some speedups by verifying multiple signatures at once do require random numbers.
8.3. Use of Contexts

Contexts can be used to separate uses of the protocol between different protocols (which is very hard to reliably do otherwise) and between different uses within the same protocol. However, the following SHOULD be kept in mind when using this facility:

The context SHOULD be a constant string specified by the protocol using it. It SHOULD NOT incorporate variable elements from the message itself.

Contexts SHOULD NOT be used opportunistically, as that kind of use is very error prone. If contexts are used, one SHOULD require all signature schemes available for use in that purpose support contexts.

Contexts are an extra input, which percolate out of APIs; as such, even if the signature scheme supports contexts, those may not be available for use. This problem is compounded by the fact that many times the application is not invoking the signing and verification functions directly but via some other protocol.

8.4. Signature Malleability

Some systems assume signatures are not malleable: that is, given a valid signature for some message under some key, the attacker can’t produce another valid signature for the same message and key.

Ed25519 and Ed448 signatures are not malleable due to the verification check that decoded $S$ is smaller than $l$. Without this check, one can add a multiple of $l$ into a scalar part and still pass signature verification, resulting in malleable signatures.

8.5. Choice of Signature Primitive

Ed25519 and Ed25519ph have a nominal strength of 128 bits, whereas Ed448 and Ed448ph have the strength of 224. While the lower strength is sufficient for the foreseeable future, the higher level brings some defense against possible future cryptographic advances. Both are demolished by quantum computers just about the same.

The Ed25519ph and Ed448ph variants are prehashed. This is mainly useful for interoperability with legacy APIs, since in most of the cases, either the amount of data signed is not large or the protocol is in the position to do digesting in ways better than just prehashing (e.g., tree hashing or splitting the data). The
Prehashing also makes the functions greatly more vulnerable to weaknesses in hash functions used. These variants SHOULD NOT be used.

Ed25519ctx and Ed448 have contexts. However, this is balanced by the problems noted in Section 8.3 about contexts.

On the implementation front, Ed25519 is widely implemented and has many high-quality implementations. The others have much worse support.

In summary, if a high 128-bit security level is enough, use of Ed25519 is RECOMMENDED; otherwise, Ed448 is RECOMMENDED.

8.6. Mixing Different Prehashes

The schemes described in this document are designed to be resistant to mixing prehashes. That is, it is infeasible to find a message that verifies using the same signature under another scheme, even if the original signed message was chosen. Thus, one can use the same key pair for Ed25519, Ed25519ctx, and Ed25519ph and correspondingly with Ed448 and Ed448ph.

The "SigEd25519 no Ed25519 collisions" constant is chosen to be a textual string such that it does not decode as a point. Because the inner hash input in the Ed25519 signature always starts with a valid point, there is no way trivial collision can be constructed. In the case of seed hash, trivial collisions are so unlikely, even with an attacker choosing all inputs, that it is much more probable that something else goes catastrophically wrong.

8.7. Signing Large Amounts of Data at Once

Avoid signing large amounts of data at once (where "large" depends on the expected verifier). In particular, unless the underlying protocol does not require it, the receiver MUST buffer the entire message (or enough information to reconstruct it, e.g., compressed or encrypted version) to be verified.

This is needed because most of the time, it is unsafe to process unverified data, and verifying the signature makes a pass through the whole message, causing ultimately at least two passes through.

As an API consideration, this means that any Initialize Update Finalize (IFU) verification interface is prone to misuse.
It is a bad idea to modify Ed25519 or Ed448 signing to be able to create valid Ed25519/Ed448 signatures using an IUF interface with only constant buffering. Pretty much any error in such would cause catastrophic security failure.

8.8. Multiplication by Cofactor in Verification

The given verification formulas for both Ed25519 and Ed448 multiply points by the cofactor. While this is not strictly necessary for security (in fact, any signature that meets the non-multiplied equation will satisfy the multiplied one), in some applications it is undesirable for implementations to disagree about the exact set of valid signatures. Such disagreements could open up, e.g., fingerprinting attacks.

8.9. Use of SHAKE256 as a Hash Function

Ed448 uses SHAKE256 as a hash function, even if SHAKE256 is specifically defined not to be a hash function.

The first potentially troublesome property is that shorter outputs are prefixes of longer ones. This is acceptable because output lengths are fixed.

The second potentially troublesome property is failing to meet standard hash security notions (especially with preimages). However, the estimated 256-bit security level against collisions and preimages is sufficient to pair with a 224-bit level elliptic curve.

9. References

9.1. Normative References


9.2. Informative References


Appendix A. Ed25519/Ed448 Python Library

Below is an example implementation of Ed25519/Ed448 written in Python; version 3.2 or higher is required.

Note: This code is not intended for production. Although it should produce correct results for every input, it is slow and makes no attempt to avoid side-channel attacks.

```python
import hashlib;
import os;

#Compute candidate square root of x modulo p, with p = 3 (mod 4).
def sqrt4k3(x,p): return pow(x,(p + 1)//4,p)

#Compute candidate square root of x modulo p, with p = 5 (mod 8).
def sqrt8k5(x,p):
    y = pow(x,(p+3)//8,p)
    #If the square root exists, it is either y or y*2^(p-1)/4.
    if (y * y) % p == x % p: return y
    else:
        z = pow(2,(p - 1)//4,p)
        return (y * z) % p

#Decode a hexadecimal string representation of the integer.
def hexi(s): return int.from_bytes(bytes.fromhex(s),byteorder="big")

#Rotate a word x by b places to the left.
def rol(x,b): return ((x << b) | (x >> (64 - b))) & (2**64-1)

#From little endian.
def from_le(s): return int.from_bytes(s, byteorder="little")

#Do the SHA-3 state transform on state s.
def sha3_transform(s):
    ROTATIONS = [0,1,62,28,27,36,44,6,55,20,3,10,43,25,39,41,45,15,
                 21,8,18,2,61,56,14]
    PERMUTATION = [1,6,9,22,14,20,2,12,13,19,23,15,4,24,21,8,16,5,3,
                   18,17,11,7,10]
    RC = [0x0000000000000001,0x0000000000000082,0x800000000000008a,
          0x8000000000000080,0x0000000000000080b,0x0000000000000001,
          0x8000000000000080,0x8000000000000009,0x0000000000000080a,
          0x0000000000000088,0x00000000000000809,0x0000000000000000a,
          0x0000000000000080,0x8000000000000008b,0x8000000000000089,
          0x80000000000000803,0x80000000000000802,0x8000000000000080,
          0x0000000000000080a,0x8000000000000000a,0x8000000000000081,
          0x8000000000000080,0x8000000000000000,0x8000000000000001
```
for rnd in range(0,24):
    #AddColumnParity (Theta)
    c = [0]*5;
    d = [0]*5;
    for i in range(0,25): c[i%5] ^= s[i]
    for i in range(0,5): d[i] = c[(i+4)%5] ^ rol(c[(i+1)%5],1)
    for i in range(0,25): s[i] ^= d[i%5]
    #RotateWords (Rho)
    for i in range(0,25): s[i] = rol(s[i],ROTATIONS[i])
    #PermuteWords (Pi)
    t = s[PERMUTATION[0]]
    for i in range(0,len(PERMUTATION)-1):
        s[PERMUTATION[i]] = s[PERMUTATION[i+1]]
    s[PERMUTATION[-1]] = t;
    #NonlinearMixRows (Chi)
    for i in range(0,25,5):
        t = [s[i],s[i+1],s[i+2],s[i+3],s[i],s[i+1]]
        for j in range(0,5): s[i+j] = t[j] ^ ((~t[j+1]) & (t[j+2]))
    #AddRoundConstant (Iota)
    s[0] ^= RC[rnd]

#Reinterpret octet array b to word array and XOR it to state s.
def reinterpret_to_words_and_xor(s,b):
    for j in range(0,len(b)//8):
        s[j] ^= from_le(b[8*j:8])

#Reinterpret word array w to octet array and return it.
def reinterpret_to_octets(w):
    mp = bytearray()
    for j in range(0,len(w)):
        mp += w[j].to_bytes(8, byteorder="little")
    return mp
def sha3_raw(msg, r_w, o_p, e_b):
    r_b = 8 * r_w
    s = 0]*25
    # Handle whole blocks.
    idx = 0
    blocks = len(msg) // r_b
    for i in range(0, blocks):
        reinterpret_to_words_and_xor(s, msg[idx:][:r_b])
        idx += r_b
    sha3_transform(s)
    # Handle last block padding.
    m = bytearray(msg[idx:])
    m.append(o_p)
    while len(m) < r_b: m.append(0)
    m[len(m)-1]|=128
    # Handle padded last block.
    reinterpret_to_words_and_xor(s, m)
    sha3_transform(s)
    # Output.
    out = bytearray()
    while len(out) < e_b:
        out += reinterpret_to_octets(s[:r_w])
    sha3_transform(s)
    return out[:e_b]

# Implementation of SHAKE256 functions.
def shake256(msg, olen):
    return sha3_raw(msg, 17, 31, olen)
A (prime) field element.

class Field:
    #Construct number x (mod p).
    def __init__(self,x,p):
        self.__x=x%p
        self.__p=p
    #Check that fields of self and y are the same.
    def __check_fields(self,y):
        if type(y) is not Field or self.__p!=y.__p:
            raise ValueError("Fields don’t match")
    #Field addition. The fields must match.
    def __add__(self,y):
        self.__check_fields(y)
        return Field(self.__x+y.__x,self.__p)
    #Field subtraction. The fields must match.
    def __sub__(self,y):
        self.__check_fields(y)
        return Field(self.__p+self.__x-y.__x,self.__p)
    #Field negation.
    def __neg__(self):
        return Field(self.__p-self.__x,self.__p)
    #Field multiplication. The fields must match.
    def __mul__(self,y):
        self.__check_fields(y)
        return Field(self.__x*y.__x,self.__p)
    #Field division. The fields must match.
    def __truediv__(self,y):
        return self*y.inv()
    #Field inverse (inverse of 0 is 0).
    def inv(self):
        return Field(pow(self.__x,self.__p-2,self.__p),self.__p)
    #Field square root. Returns none if square root does not exist.
    #Note: not presently implemented for p mod 8 = 1 case.
    def sqrt(self):
        #Compute candidate square root.
        if self.__p%4==3: y=sqrt4k3(self.__x,self.__p)
        elif self.__p%8==5: y=sqrt8k5(self.__x,self.__p)
        else: raise NotImplementedError("sqrt(_,8k+1)")
        _y=Field(y,self.__p);
        #Check square root candidate valid.
        return _y if _y*_y==self else None
    #Make the field element with the same field as this, but
    #with a different value.
    def make(self,ival): return Field(ival,self.__p)
    #Is the field element the additive identity?
    def iszero(self): return self.__x==0
    #Are field elements equal?
    def __eq__(self,y): return self.__x==y.__x and self.__p==y.__p
# Are field elements not equal?
def __ne__(self, y):
    return not (self == y)

# Serialize number to b-1 bits.
def tobytes(self, b):
    return self.__x.to_bytes(b // 8, byteorder="little")

# Unserialize number from bits.
def frombytes(self, x, b):
    rv = from_le(x) % (2 ** (b - 1))
    return Field(rv, self.__p) if rv < self.__p else None

# Compute sign of number, 0 or 1. The sign function has the following property:
# sign(x) = 1 - sign(-x) if x != 0.
def sign(self):
    return self.__x % 2

# A point on (twisted) Edwards curve.
class EdwardsPoint:
    # base_field = None
    # x = None
    # y = None
    # z = None
    def initpoint(self, x, y):
        self.x = x
        self.y = y
        self.z = self.base_field.make(1)
    
def decode_base(self, s, b):
        # Check that point encoding is the correct length.
        if len(s) != b // 8:
            return (None, None)
        # Extract signbit.
        xs = s[(b - 1) // 8] >> ((b - 1) & 7)
        # Decode y. If this fails, fail.
        y = self.base_field.frombytes(s, b)
        if y is None:
            return (None, None)
        # Try to recover x. If it does not exist, or if zero and xs are wrong, fail.
        x = self.solve_x2(y).sqrt()
        if x is None or (x.iszero() and xs != x.sign()):
            return (None, None)
        # If sign of x isn’t correct, flip it.
        if x.sign() != xs:
            x = -x
        # Return the constructed point.
        return (x, y)
    
def encode_base(self, b):
        xp, yp = self.x / self.z, self.y / self.z
        # Encode y.
        s = bytearray(yp.tobytes(b))
        # Add sign bit of x to encoding.
        if xp.sign() != 0:
            s[(b - 1) // 8] |= 1 << (b - 1) & 8
        return s
def __mul__(self, x):
    r = self.zero Elem()
    s = self
    while x > 0:
        if (x%2) > 0:
            r = r + s
        s = s.double()
        x = x // 2
    return r

# Check that two points are equal.
def __eq__(self, y):
    # Need to check x1/z1 == x2/z2 and similarly for y, so cross
    # multiply to eliminate divisions.
    xn1 = self.x * y.z
    xn2 = y.x * self.z
    yn1 = self.y * y.z
    yn2 = y.y * self.z
    return xn1 == xn2 and yn1 == yn2

# Check if two points are not equal.
def __ne__(self, y):
    return not (self == y)

# A point on Edwards25519.
class Edwards25519Point(EdwardsPoint):
    # Create a new point on the curve.
    base_field = Field(1, 2**255-1)
    d = -base_field.make(121665) / base_field.make(121666)
    f0 = base_field.make(0)
    f1 = base_field.make(1)
    xb = base_field.make(hexi("216936D3CD6E53FEC0A4E231FDD6DC5C692CC76"+
                             "09525A7B2C9562D608F25D51A"))
    yb = base_field.make(hexi("66666666666666666666666666666666666666666666666666666666666666666666666666666666666"))
    # The standard base point.
    @staticmethod
    def stdbase():
        return Edwards25519Point(xb, yb)
    def __init__(self, x, y):
        # Check the point is actually on the curve.
        if y * y - x * x != self.f1 + self.d * x * x * y * y:
            raise ValueError("Invalid point")
        self.initpoint(x, y)
        self.t = x * y
    # Decode a point representation.
    def decode(self, s):
        x, y = self.decode_base(s, 256);
        return Edwards25519Point(x, y) if x is not None else None
# Encode a point representation.
def encode(self):
    return self.encode_base(256)

# Construct a neutral point on this curve.
def zero_elem(self):
    return Edwards25519Point(self.f0, self.f1)

# Solve for x^2.
def solve_x2(self, y):
    return ((y*y-self.f1)/(self.d*y*y+self.f1))

# Point addition.
def __add__(self, y):
    # The formulas are from EFD.
    tmp = self.zero_elem()
    zcp = self.z*y.z
    A = (self.y-self.x)*(y.y-y.x)
    B = (self.y+self.x)*(y.y+y.x)
    C = (self.d+self.d)*self.t*y.t
    D = zcp+zcp
    E, H = B-A, B+A
    F, G = D-C, D+C
    tmp.x, tmp.y, tmp.z, tmp.t = E*F, G*H, F*G, E*H
    return tmp

# Point doubling.
de double(self):
    # The formulas are from EFD (with assumption a=-1 propagated).
    tmp = self.zero_elem()
    A = self.x*self.x
    B = self.y*self.y
    Ch = self.z*self.z
    C = Ch+Ch
    H = A+B
    xys = self.x+self.y
    E = H-xys*xys
    G = A-B
    F = C+G
    tmp.x, tmp.y, tmp.z, tmp.t = E*F, G*H, F*G, E*H
    return tmp

# Order of basepoint.
def l(self):
    return hexi("1000000000000000000000000000000014def9dea2f79cd"+
                 "65812631a5cf5d3ed")

# The logarithm of cofactor.
def c(self): return 3

# The highest set bit
edef n(self): return 254

# The coding length
def b(self): return 256
# Validity check (for debugging)
def is_valid_point(self):
x, y, z, t = self.x, self.y, self.z, self.t
x2 = x * x
y2 = y * y
z2 = z * z
lhs = (y2 - x2) * z2
rhs = z2 * z2 + self.d * x2 * y2
assert(lhs == rhs)
assert(t * z == x * y)

# A point on Edwards448.
class Edwards448Point(EdwardsPoint):
    # Create a new point on the curve.
    base_field = Field(1, 2**448 - 2**224 - 1)
d = base_field.make(-39081)
f0 = base_field.make(0)
f1 = base_field.make(1)
xb = base_field.make(hexi("4F1970C66BED0DED221D15A622BF36DA9E14657"+)"0470F1767EAE6DE324A3D3A46412AE1AF72AB66511433B80E18B00938E26"+)"26A82BC70CC05E")
yb = base_field.make(hexi("693F46716EB6BC248876203756C9C7624BEA737"+)"36CA3984087789C1E05A0C2D73AD3FF1CE67C39C4FDBD132C4ED7C8AD98"+)"08795BF230FA14")
    # The standard base point.
    @staticmethod
def stdbase():
        return Edwards448Point(Edwards448Point.xb, Edwards448Point.yb)
def __init__(self, x, y):
    # Check that the point is actually on the curve.
    if y * y + x * x != self.f1 + self.d * x * y * y:
        raise ValueError("Invalid point")
    self.initpoint(x, y)
    # Decode a point representation.
def decode(self, s):
        x, y = self.decode_base(s, 456);
        return Edwards448Point(x, y) if x is not None else None
    # Encode a point representation.
def encode(self):
        return self.encode_base(456)
    # Construct a neutral point on this curve.
def zero_elem(self):
        return Edwards448Point(self.f0, self.f1)
    # Solve for x^2.
def solve_x2(self, y):
        return ((y * y - self.f1) / (self.d * y * y - self.f1))
#Point addition.
def __add__(self,y):
    #The formulas are from EFD.
    tmp=self.zero_elem()
    xcp,ycp,zcp=self.x*y.x,self.y*y.y,self.z*y.z
    B=zcp*zcp
    E=self.d*xcp*ycp
    F,G=B-E,B+E
    tmp.x=zcp*F*((self.x+self.y)*(y.x+y.y)-xcp-ycp)
    tmp.y,tmp.z=zcp*G*(ycp-xcp),F*G
    return tmp

#Point doubling.
def double(self):
    #The formulas are from EFD.
    tmp=self.zero_elem()
    x1s,y1s,z1s=self.x*self.x,self.y*self.y,self.z*self.z
    xys=self.x+self.y
    F=x1s+y1s
    J=F-(z1s+z1s)
    tmp.x,tmp.y,tmp.z=(xys*xys-x1s-y1s)*J,F*(x1s-y1s),F*J
    return tmp

#Order of basepoint.
def l(self):
    return hexi("3fffffffffffffffffffffffffffffffffffffffffffffffffffffffff"+\
                "fffffffff7cca23e9c44ed5b49aed63690216cc2728dc58f552378c2"+\
                "92ab3844f3")

#The logarithm of cofactor.
def c(self): return 2

#The highest set bit.
def n(self): return 447

#The coding length.
def b(self): return 456

#Validity check (for debugging).
def is_valid_point(self):
    x,y,z=self.x,self.y,self.z
    x2=x*x
    y2=y*y
    z2=z*z
    lhs=(x2+y2)*z2
    rhs=z2*z2+self.d*x2*y2
    assert(lhs == rhs)
# Simple self-check.
def curve_self_check(point):
    p = point
    q = point.zero_elem()
    z = q
    l = p.l() + 1
    p.is_valid_point()
    q.is_valid_point()
    for i in range(0, point.b()):
        if (l >> i) & 1 != 0:
            q = q + p
            q.is_valid_point()
            p = p.double()
            p.is_valid_point()
    assert q.encode() == point.encode()
    assert q.encode() != p.encode()
    assert q.encode() != z.encode()

# Simple self-check.
def self_check_curves():
    curve_self_check(Edwards25519Point.stdbase())
    curve_self_check(Edwards448Point.stdbase())

# PureEdDSA scheme.
# Limitation: only b mod 8 = 0 is handled.
class PureEdDSA:
    # Create a new object.
    def __init__(self, properties):
        self.B = properties['B']
        self.H = properties['H']
        self.l = self.B.l()
        self.n = self.B.n()
        self.b = self.B.b()
        self.c = self.B.c()
    # Clamp a private scalar.
    def __clamp(self, a):
        _a = bytearray(a)
        for i in range(0, self.c):
            _a[i // 8] &= ~(1 << (i % 8))
        _a[self.n // 8] = 1 << (self.n % 8)
        for i in range(self.n + 1, self.b):
            _a[i // 8] &= ~(1 << (i % 8))
        return _a
    # Generate a key. If privkey is None, a random one is generated.
    # In any case, the (privkey, pubkey) pair is returned.
    def keygen(self, privkey):
        # If no private key data is given, generate random.
        if privkey is None:
            privkey = os.urandom(self.b // 8)
def sign(self, privkey, pubkey, msg, ctx, hflag):
    # Expand key.
    khash = self.H(privkey, None, None)
    a = from_le(self.__clamp(khash[:self.b//8]))
    # Return the key pair (public key is A=Enc(aB).
    return privkey, (self.B*a).encode()

def verify(self, pubkey, msg, sig, ctx, hflag):
    # Sanity-check sizes.
    if len(sig) != self.b//4: return False
    if len(pubkey) != self.b//8: return False
    # Split signature into R and S, and parse.
    Rraw, Sraw = sig[:self.b//8], sig[self.b//8:]
    R, S = self.B.decode(Rraw), from_le(Sraw)
    # Parse public key.
    A = self.B.decode(pubkey)
    # Check parse results.
    if (R is None) or (A is None) or S >= self.l: return False
    # Calculate h.
    h = from_le(self.H(Rraw+pubkey+msg, ctx, hflag)) % self.l
    # Calculate left and right sides of check eq.
    rhs = R + (A*h)
    lhs = self.B*S
    for i in range(0, self.c):
        lhs = lhs.double()
        rhs = rhs.double()
    # Check eq. holds?
    return lhs == rhs

def Ed25519_inthash(data, ctx, hflag):
    if (ctx is not None and len(ctx) > 0) or hflag:
        raise ValueError("Contexts/hashes not supported")
    return hashlib.sha512(data).digest()
# The base PureEdDSA schemes.

def Ed25519ctx_inthash(data, ctx, hflag):
    dompfx = b"
    if ctx is not None:
        if len(ctx) > 255: raise ValueError("Context too big")
        dompfx = PREFIX + bytes([1 if hflag else 0, len(ctx)]) + ctx
    return hashlib.sha512(dompfx + data).digest()

pEd25519ctx = PureEdDSA({
    "B": Edwards25519Point.stdbase(),
    "H": Ed25519ctx_inthash
})

def Ed448_inthash(data, ctx, hflag):
    dompfx = b"
    if ctx is not None:
        if len(ctx) > 255: raise ValueError("Context too big")
        dompfx = b"SigEd448" + bytes([1 if hflag else 0, len(ctx)]) + ctx
    return shake256(dompfx+data, 114)

pEd448 = PureEdDSA({
    "B": Edwards448Point.stdbase(),
    "H": Ed448_inthash
})

# EdDSA scheme.
class EdDSA:
    # Create a new scheme object, with the specified PureEdDSA base
    # scheme and specified prehash.
    def __init__(self, pure_scheme, prehash):
        self.__pflag = True
        self.__pure = pure_scheme
        self.__prehash = prehash
        if self.__prehash is None:
            self.__prehash = lambda x,y: x
        self.__pflag = False

    def keygen(self, privkey):
        return self.__pure.keygen(privkey)
# Sign message msg using specified key pair.
def sign(self, privkey, pubkey, msg, ctx=None):
    if ctx is None: ctx=b""
    return self.__pure.sign(privkey, pubkey, self.__prehash(msg, ctx),
                             ctx, self.__pflag)

# Verify signature sig on message msg using public key pubkey.
def verify(self, pubkey, msg, sig, ctx=None):
    if ctx is None: ctx=b""
    return self.__pure.verify(pubkey, self.__prehash(msg, ctx), sig,
                              ctx, self.__pflag)

def Ed448ph_prehash(data, ctx):
    return shake256(data, 64)

#Our signature schemes.
Ed25519 = EdDSA(pEd25519, None)
Ed25519ctx = EdDSA(pEd25519ctx, None)
Ed25519ph = EdDSA(pEd25519ctx, lambda x, y: hashlib.sha512(x).digest())
Ed448 = EdDSA(pEd48, None)
Ed448ph = EdDSA(pEd48, Ed448ph_prehash)

def eddsa_obj(name):
    if name == "Ed25519": return Ed25519
    if name == "Ed25519ctx": return Ed25519ctx
    if name == "Ed25519ph": return Ed25519ph
    if name == "Ed448": return Ed448
    if name == "Ed448ph": return Ed448ph
    raise NotImplementedError("Algorithm not implemented")

Appendix B. Library Driver

Below is a command-line tool that uses the library above to perform
computations for interactive use or for self-checking.

import sys
import binascii

from eddsa2 import Ed25519

def munge_string(s, pos, change):
    return (s[:pos] +
            int.to_bytes(s[pos] ^ change, 1, "little") +
            s[pos+1:])
# Read a file in the format of
# http://ed25519.cr.yp.to/python/sign.input
lineno = 0
while True:
    line = sys.stdin.readline()
    if not line:
        break
    lineno = lineno + 1
    print(lineno)
    fields = line.split(":")
    secret = (binascii.unhexlify(fields[0]))[:32]
    public = binascii.unhexlify(fields[1])
    msg = binascii.unhexlify(fields[2])
    signature = binascii.unhexlify(fields[3])[:64]

    privkey, pubkey = Ed25519.keygen(secret)
    assert public == pubkey
    assert signature == Ed25519.sign(privkey, pubkey, msg)
    assert Ed25519.verify(public, msg, signature)
    if len(msg) == 0:
        bad_msg = b"x"
    else:
        bad_msg = munge_string(msg, len(msg) // 3, 4)
    assert not Ed25519.verify(public, bad_msg) // 3, 4)
    assert not Ed25519.verify(public, msg, munge_string(signature, 20, 8))
    assert not Ed25519.verify(public, msg, munge_string(signature, 40, 16))
Acknowledgements

EdDSA and Ed25519 were initially described in a paper due to Daniel J. Bernstein, Niels Duif, Tanja Lange, Peter Schwabe, and Bo-Yin Yang. The Ed448 curve is due to Mike Hamburg.

An earlier draft version of this document was coauthored by Niels Moeller.

Feedback on this document was received from Werner Koch, Damien Miller, Bob Bradley, Franck Rondepierre, Alexey Melnikov, Kenny Paterson, and Robert Edmonds.

The Ed25519 test vectors were double checked by Bob Bradley using three separate implementations (one based on TweetNaCl and two different implementations based on code from SUPERCOP).

Authors’ Addresses

Simon Josefsson
SJD AB
Email: simon@josefsson.org
URI: http://josefsson.org/

Ilari Liusvaara
Independent
Email: ilariliusvaara@welho.com